SOLUTION SET

J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC TEST - 05 - SET 2

DURATION - 1 $^{1}/_{2}$ HR MARKS - 40

SECTION - I

Q1. Attempt any THREE of the following (2 marks each) (6 marks)

Q1

01. $\int \frac{e^x + 1}{e^x + x} \, dx$

SOLUTION

PUT

$e^x + x = t$

$e^x + 1 \, . \, dx = dt$

NOW THE SUM IS

$= \int \frac{1}{t} \, dt$

$= \log | t | + c$

RESUBS.

$= \log | e^x + x | + c$

02. $\int \frac{\sec^2 x}{\tan^2 x + 4} \, dx$

SOLUTION

PUT $\tan x = t$

$\sec^2 x \, . \, dx = dt$

THE SUM IS

$= \int \frac{1}{t^2 + 4} \, dt$

$= \int \frac{1}{t^2 + 2^2.} \, dt$

$= \frac{1}{a} \tan^{-1} \frac{t}{a} + c$

$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$

Resubs.

$= \frac{1}{2} \tan^{-1} \left(\frac{\tan x}{2} \right) + c$

Q1

03. $\int e^x (1 + \tan^2 x + \tan x)\, dx$

SOLUTION

$$= \int e^x \tan x + \sec^2 x \quad dx$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

HENCE THE SUM IS

$$= \int e^x \left[f(x) + f'(x) \right] dx$$

$$= e^x f(x) + c$$

$$= e^x \tan x + c$$

04. $\int \frac{1}{\sqrt{9 - 8x + x^2}}\, dx$

SOLUTION

$$= \int \frac{1}{\sqrt{9 - (x^2 - 8x)}}\, dx$$

$$= \int \frac{1}{\sqrt{9 - (x^2 - 8x + 16) + 16}}\, dx$$

$$= \int \frac{1}{\sqrt{25 - (x - 4)^2}}\, dx$$

$$= \int \frac{1}{\sqrt{5^2 - (x - 4)^2}}\, dt$$

$$= \sin^{-1} \frac{x}{a} + c$$

$$= \sin^{-1} \left(\frac{x - 4}{5} \right) + c$$

Q2. Attempt any TWO of the following (3 marks each) **(6 marks)**

Q2

01. $\int \frac{\tan x}{\sec x + \tan x}\, dx$

SOLUTION

$$= \int \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}\, dx$$

$$= \int \frac{\sin x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x}\, dx$$

$$= \int \frac{\sin x - \sin^2 x}{1 - \sin^2 x}\, dx$$

$$= \int \frac{\sin x - \sin^2 x}{\cos^2 x} dx$$

$$= \int \left(\frac{\sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\right) dx$$

$$= \int (\sec x \cdot \tan x - \tan^2 x)\, dx$$

$$= \int \left((\sec x \cdot \tan x - (\sec^2 x - 1)\right) dx$$

$$= \int (\sec x \cdot \tan x - \sec^2 x + 1\, dx$$

$$= \sec x - \tan x + x\, c$$

02. $\int \log(1+x^2)\, dx$

SOLUTION

$$= \int \log(1 + x^2).\, 1 dx$$

$$= \log(1 + x^2) \int 1\, dx - \int \left(\frac{d}{dx} \log(1 + x^2) \int 1\, dx\right) dx$$

$$= \log(1 + x^2) \cdot x - \int \frac{2x}{1 + x^2.} \cdot x\, dx$$

$$= x. \log(1 + x^2) - 2 \int \frac{x^2}{1 + x^2.}\, dx$$

$$= \quad x.\log(1+x^2) \quad - \quad 2\int \frac{1+x^2-1}{1+x^2}.\,dx$$

$$= \quad x.\log(1+x^2) \quad - \quad 2\int \left(1 \quad - \frac{1}{1+x^2}\right) dx$$

$$= \quad x.\log(1+x^2) \quad - \quad 2 \quad \left(x \quad - \quad \tan^{-1}x\right) + \quad c$$

$$= \quad x.\log(1+x^2) \quad - \quad 2x \quad + \quad 2\tan^{-1}x \quad + \quad c$$

Q3. Attempt any TWO of the following (4 marks each) **(8 marks)**

Q3

01. $\int \frac{x^2}{x^4 + 5x^2 + 6}\, dx$

SOLUTION

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{A}{x^2 + 2} + \frac{B}{x^2 + 3}$$

$x^2 = t$ (say)

$$\frac{t}{(t + 2)(t + 3)} = \frac{A}{t + 2} + \frac{B}{t + 3}$$

$$t = A(t + 3) + B(t + 2)$$

Put $t = -3$

$$-3 = B(-3 + 2)$$

$$-3 = B(-1) \qquad \therefore B = 3$$

Put $t = -2$

$$-2 = A(-2 + 3)$$

$$-2 = A(1) \qquad \therefore A = -2$$

THEREFORE

$$\frac{t}{(t + 2)(t + 3)} = \frac{-2}{t + 2} + \frac{3}{t + 3}$$

HENCE

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}$$

BACK IN THE SUM

$$= \int \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}\, dx$$

$$= \int \frac{-2}{x^2 + \sqrt{2}^2} + \frac{3}{x^2 + \sqrt{3}^2}\, dx$$

$$= -2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + 3 \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= -\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

Q3

02. $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}\,dx$

SOLUTION

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

USING $\int_a^b f(x)\,dx = \int_b^b f(a+b-x)\,dx$

WE CHANGE 'x' TO '$\pi/6 + \pi/3 - x$' i.e.

WE CHANGE 'x' TO '$\pi/2 - x$'

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \textbf{... (2)}$$

(1) + (2)

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$2I = \left[x\right]_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$$2I = \frac{2\pi - \pi}{6}$$

$$I = \frac{\pi}{12}$$

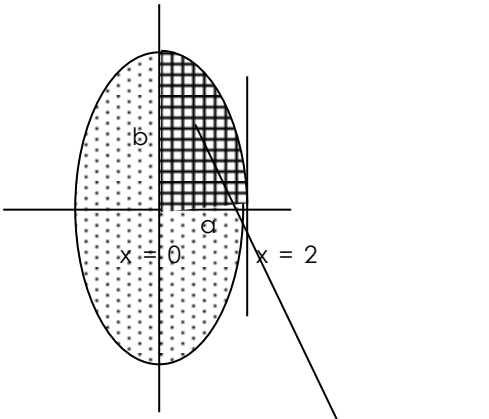
03. Find the area of the ellipse : $\frac{x^2}{4} + \frac{y^2}{25} = 1$

SOLUTION

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

$a^2 = 4 \qquad b^2 = 25$

$a = 2 \qquad b = 5$



$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

$$\frac{y^2}{25} = 1 - \frac{x^2}{4}$$

$$\frac{y^2}{25} = \frac{4 - x^2}{4}$$

$$y^2 = \frac{25}{4}(4 - x^2)$$

$$y = \frac{5\sqrt{4 - x^2}}{2}$$

Area of Ellipse

$$= 4\int_0^2 y \, dx \quad \text{..... BY SYMMETRY}$$

$$= 4\int_0^2 \frac{5\sqrt{4 - x^2}}{2} dx$$

$$= 10\int_0^2 \sqrt{2^2 - x^2} \, dx$$

$$= 10\left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 10\left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 10\left\{\left[\frac{2}{2}\sqrt{2^2 - 2^2} + 2\sin^{-1}\left(\frac{2}{2}\right)\right] - \left[\frac{0}{2}\sqrt{2^2 - 0^2} + 2\sin^{-1}\left(\frac{0}{2}\right)\right]\right\}$$

$$= 10\left[0 + 2\sin^{-1}(1)\right] - \left[0 + 2\sin^{-1}(0)\right]$$

$$= 10\left(2 \times \frac{\pi}{2}\right)$$

$$= 10\pi \text{ sq. units}$$

SECTION - II

Q4. **Attempt any THREE of the following** (2 marks each) **(6 marks)**

Q4

01. Compute Age – Specific Death rate for the following data

AGE GROUP	POPULATION IN 000	NO. OF DEATHS	SDR = $\frac{D}{P}$
0 – 10	25	50	$\frac{50}{25}$ = 2
10 – 30	30	90	$\frac{90}{30}$ = 3
30 – 45	40	160	$\frac{160}{40}$ = 4
45 – 70	20	100	$\frac{100}{20}$ = 5

02. The time (in hours) required to perform the printing and binding operations (in that order) for each book is given in the following table :

Books	:	I	II	III	IV	V
Printing Machine M_1	:	3	7	4	5	7
Binding Machine M_2	:	6	2	7	3	4

Find the sequence that minimizes the total elapsed time (in hours) to complete the work

Min time = 2 on job II on machine M_2 . Place the job at the end of the sequence

				II

Next min time = 3 on job I on machine M_1 & on job IV on machine M_2 . Place the job I at the start of the sequence & job IV at the end of the sequence before II

I			**IV**	**II**

Next min time = 4 on job III on machine M_1 & on job V on machine M_2 . Place the job III at the start of the sequence after I & job V at the end of the sequence before IV

I	**III**	**V**	**IV**	**II**

OPTIMAL SEQUENCE

I	**III**	**V**	**IV**	**II**

Q4

03. Determine l_3 and l_4 given that $l_2 = 80$; $d_2 = 5$; $q_3 = 0.20$

STEP 1 :

$dx = lx - lx+1$

$d_2 = l_2 - l_3$

$5 = 80 - l_3$

$\boldsymbol{l_3 = 75}$

STEP 2 :

$px = 1 - qx$

$p_3 = 1 - q_3$

$= 1 - 0.2$

$\boldsymbol{p_3 = 0.8}$

STEP 3 :

$$px = \frac{lx+1}{lx}$$

$$p_3 = \frac{l_4}{l_3}$$

$$0.8 = \frac{l_4}{75}$$

$\boldsymbol{l_4 = 60}$

04.

Age Group	Population	No. of deaths
0 – 20	40000	350
20 – 65	65000	650
65 & above	15000	x
	$\Sigma P = 120000$	$\Sigma D = 1000 + x$

$$CDR = \frac{\Sigma D \times 1000}{\Sigma P}$$

$$13.4 = \frac{1000 + x}{120\cancel{000}} \times \cancel{1000}$$

$1608 = 1000 + x$

$x = 608$

Q5. Attempt any TWO of the following (3 marks each) **(6 marks)**

Q5

01.

Age Group (Years)	City A		City B	
	POPULATION	NO. OF DEATHS	POPULATION	NO. OF DEATHS
0 – 10	800	32	900	12
10 – 25	3000	12	1500	8
25 – 65	4800	48	4500	38
65 & above	1400	42	600	30
	ΣP =10000	ΣD = 134	ΣP =7500	ΣD = 88

CDR (TOWN I) $= \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{134}{10000} \times 1000$

$= 13.4$

(DEATHS PER THOUSAND)

CDR (TOWN II) $= \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{88}{7500} \times 1000$

$= \frac{880}{75}$

$= 11.73$

(DEATHS PER THOUSAND)

LOG CALC

2.9445
– 1.8751
AL 1.0694
11.73

COMMENT : CDR(CITY B) < CDR(CITY A) . HENCE CITY B IS HEALTHIER THAN CITY A

02. COMPLETE THE TABLE

Q5

x	l_x	d_x	q_x	p_x	L_x	T_x
20	88230	?	?	?	?	?
21	79473	-	-	-	-	3205552

$d_x = l_x - l_{x+1}$

$d_{20} = l_{20} - l_{21}$

$= 88230 - 79473$

$= 8757$

$q_x = \frac{d_x}{l_x}$

$q_{20} = \frac{d_{20}}{l_{20}} = \frac{8757}{88230}$

$= 0.09924$

LOG CALC
3.9423
– 4.9456
AL $\overline{2}$.9967
0.09924

$p_x = 1 - q_x$

$p_{20} = 1 - q_{20}$

$= 1 - 0.09924$

$= 0.90076$

$L_x = \frac{l_x + l_{x+1}}{2}$

$L_{20} = \frac{l_{20} + l_{21}}{2}$

$= \frac{88230 + 79473}{2}$

$= 83851.5$

$= 83852$

$T_{x+1} = T_x - L_x$

$T_{21} = T_{20} - L_{20}$

$3205552 = T_{20} - 83852$

$T_{20} = 3205552 + 83852$

$T_{20} = 3289404$

ans :

x	**l_x**	**d_x**	**q_x**	**p_x**	**L_x**	**T_x**
20	88230	**8757**	**0.09924**	**0.90076**	**83852**	**3289404**
21	79473	**---**	**----**	**----**	**---**	3205552

03. Solve the minimal assignment problem

Q5

		MEN			
		A	B	C	D
	1	16	1	6	11
JOBS	2	25	10	0	10
	3	10	25	2	14
	4	15	7	14	10

15	0	5	10	Reducing the matrix using ' ROW MINIMUM'
25	10	0	10	
8	23	0	12	
8	0	7	3	

7	0	5	7	Reducing the matrix using ' COLUMN MINIMUM'
17	10	0	7	
0	23	0	9	
0	0	7	0	

7	[0]	5	7	Allocation using 'SINGLE ZERO ROW COUMN METHOD'
17	10	[0]	7	
[0]	23	~~0~~	9	Since each row now contains an assigned zero , the
~~0~~	~~0~~	7	[0]	assignment problem is complete

OPTIMAL ASSIGNMENT

A – 3 , B – 1 , C – 2 , D – 4 , MINIMUM VALUE = 10 + 1 + 0 + 10 = 21

Q6. Attempt any TWO of the following (4 marks each) **(8 marks)**

Q6

AGE x	lx	dx = lx − lx+1	qx = dx / lx	px = 1 − qx	Lx = (lx + lx+1)/2	Tx	e_x^0 = Tx / lx
0	1000	1000 − 940= 60	60/1000 = 0.06	1 − 0.06 = 0.94	940 + 30 = 970	2835	2835/1000 = 2.835
1	940	940 − 780 = 160	160/940 = 0.17	1 − 0.17 = 0.83	780 + 80 = 860	1865	1865/940 = 1.985
2	780	780 − 590 = 190	190/780 = 0.24	1 − 0.24 = 0.76	590 + 95 = 685	1005	1005/780 = 1.288
3	590	590 − 25 = 565	565/590 = 0.96	1 − 0.96 = 0.04	25+ 282.5= 307.5	320	320/590 = 0.5423
4	25	25 − 0 = 25	25/25 = 1	1 − 1 = 0	0 + 12. 5 = 12.5	12.5	12.5/25 = 0.5
5	0	----	---- ----	----	---- ----		

LOG CALCULATIONS FOR 'qx'

LOG 160 − LOG 940	LOG 190 − LOG 780	LOG 565 − LOG 590
2.2041	2.2788	2.7520
− 2.9731	− 2.8921	− 2.7709
AL $\overline{1}$.2310	AL $\overline{1}$.3867	AL $\overline{1}$.9811
0.1702	0.2436	0.9574

LOG CALCULATIONS FOR 'e_x^0'

LOG 1865 − LOG 940	LOG 1005 − LOG 780	LOG 320 − LOG 590
3.2707	3.0021	2.5051
− 2.9731	− 2.8921	− 2.7709
AL 0.2976	AL 0.1100	AL $\overline{1}$.7342
1.985	1.288	0.5423

02. Four destinations are short listed for the holiday – outing for five VIP families . For security reasons , each family is supposed to go to a different destination . The security cost pay off matrix (in crores) for their tour is given below . Find the allocations to the destinations , so as to minimize the cost . Also mention which family will have to cancel the tour

Q6

		DESTINATIONS			
		D_1	D_2	D_3	D_4
	F_1	1	6	8	1
	F_2	6	5	7	5
FAMILIES	F_3	4	2	5	3
	F_4	6	2	1	3
	F_5	3	4	3	4

SECURITY COST (IN CRORES)

1	6	8	1	0	adding a DUMMY destination D5 to
6	5	7	5	0	balance the matrix
4	2	5	3	0	
6	2	1	3	0	
3	4	3	4	0	
0	4	7	0	0	Reducing the matrix using
5	3	6	4	0	'COLUMN MINIMIZATION'
3	0	4	2	0	
5	0	0	2	0	
2	2	2	3	0	
[0]	4	7	~~0~~	~~0~~	Allocation using
5	3	6	4	[0]	'SINGLE ZERO ROW COLUMN METHOD'
3	[0]	4	2	~~0~~	
5	~~0~~	[0]	2	~~0~~	
2	2	2	3	~~0~~	
[0]	4	7	~~0~~	~~0~~	Since allocation is incomplete , we draw min.
5	3	6	4	[0] √	lines to cover the existing zero's
3	[0]	4	2	~~0~~	
5	~~0~~	[0]	2	~~0~~	
2	2	2	3	~~0~~ √	
				√	

0	4	7	0	2	MODIFY the matrix
3	1	4	2	0	- Reduce all the uncovered elements by
3	0	4	2	2	its minimum & add the same at the
5	0	0	2	2	intersection
0	0	0	1	0	

~~0~~	4	7	[0]	2	Reallocate using
3	1	4	2	[0]	'SINGLE ZERO ROW COLUMN METHOD'
3	[0]	4	2	2	
5	~~0~~	[0]	2	2	Since all rows now contain assigned zero ,
[0]	~~0~~	~~0~~	1	~~0~~	allocation is COMPLETE

OPTIMAL ASSIGNMENT

$F_1 - D_4$, $F_2 - D_5$ (DUMMY) , $F_3 - D_2$, $F_4 - D_3$, $F_5 - D_1$

MINIMUM COST = 1 + 2 + 1 + 3 = 7 (in cr.)

03.

Job	A	B	C	D	E
M_1	5	7	6	9	5
M_2	2	1	4	5	3
M_3	3	7	5	6	7

Q6

STEP 1 : Min time on M_1 = 5 ;

Max time on M_2 = 5

Min time on M_3 = 3

Min (M_1) ≥ Max (M_2) …… condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S M_A & M_B

M_A = M_1 + M_2

M_B = M_2 + M_3

Job	A	B	C	D	E
M_A	7	8	10	14	8
M_B	5	8	9	11	10

STEP 3 : OPTIMAL SEQUENCE

Min time = 5 on job A on machine M_B . Place the job at the end of the sequence

				A

Next min time = 8 on job B & E on machine M_A . Place them randomly at the start of the sequence

B	**E**			**A**

Next min time = 9 on job C on machine M_B . Place it at the end of the sequence before A

B	**E**		**C**	**A**

OPTIMAL SEQUENCE

B	**E**	**D**	**C**	**A**

STEP 4 : WORK TABLE

Job	B	E	D	C	A		total process time
M_1	7	5	9	6	5	=	32 hrs
M_2	1	3	5	4	2	=	15 hrs
M_3	7	7	6	5	3	=	28 hrs

JOBS	**M_1**		**IDLE**	**M_2**		**IDLE**	**M_3**		**IDLE**
	IN	**OUT**	**TIME**	**IN**	**OUT**	**TIME**	**IN**	**OUT**	**TIME**
						7			8
B	0	7	--	7	8	4	8	15	--
E	7	12	--	12	15	6	15	22	4
D	12	21	--	21	26	1	26	32	--
C	21	27	--	27	31	1	32	37	--
A	27	32	8	32	34	6	37	40	--

STEP 5 : Total elapsed time T = 40 hrs

Idle time on M_1 = T – (sum of processing time of all 5 jobs on M1)

= 40 – 32

= 8 hrs

Idle time on M_2 = T – (sum of processing time of all 5 jobs on M2)

= 40 – 15

= 25 hrs (CHECK – 7 + 4 + 6 + 1 + 1 + 6 = 25)

Idle time on M_3 = T – (sum of processing time of all 5 jobs on M3)

= 40 – 28

= 12 hrs (CHECK – 8 + 4 = 12)